

Math 3345, Review for Test #1

• Section 1.1

- You should know the Field Axioms for \mathbb{R} (Postulate 1). Be able to use these axioms prove basic facts about \mathbb{R} which you ordinarily take for granted, like equations (1), (2), (3), and (4) on page 3, or #6 from the problems at the end of the section.
- Know the Order Axioms for \mathbb{R} (Postulate 2). Be able to use these axioms to prove basic facts about \mathbb{R} which you ordinarily take for granted, like Example 1.2 and 1.3, equations (7), (8), and (9) on page 7. Also, look at Theorem 1.9 and some of the end-of-section problems involving inequalities, such as #5 and #8.
- Know the definition of the absolute value of a real number a . Be able to use the definition to prove facts about real numbers, such as in Remark 1.5, Theorem 1.6, and Theorem 1.7. You should also be able to solve inequalities involving absolute values, such as in #7.

• Section 1.2

- Know the Well-Ordering Principle (Postulate 3) and how the First Principle of Mathematical Induction (Theorem 1.11) follows from the Well-Ordering Principle. Be able to do proofs by induction.
- Know the Binomial Theorem. Be able to do problems involving the Binomial Theorem, like #2, #3.

• Section 1.3

- Know the definitions of the terms *bounded above/below*, *upper/lower bound*, *supremum/infimum*. Be able to find suprema/infima (see #1), and be able to prove basic facts involving these ideas (Example 1.17, Remarks 1.18 and 1.19, Theorem 1.20).
- Know the Approximation Property for Suprema (Theorem 1.20) and its counterpart for infima (#5 end-of-section problems) and how to prove them.
- Know the Completeness Axiom.
- Know and be able to prove the very important and often used result which follows from the Completeness Axiom called the Archimedean Principle.
- Know and be able to prove Theorem 1.24 (Density of the Rationals) and end-of-section problem #3 (Density of Irrationals). Additionally, be able to prove Remark 1.25.

• Section 1.4

- Know the definitions of the terms *1-1 (injective)*, *onto (surjective)*, *bijective*, and be able to prove whether or not functions have these properties. See #1.
- Know what the inverse of a function is and how to find it. Be able to prove properties pertaining to inverses of functions. See #1.
- Know what the image and preimage of a set under a function f is. See #4.

• Section 2.1

- Know the definition of *convergence* of a sequence. Be able to prove that a limit exists. See #1 and #2.
- Be able to prove when a sequence does not converge. See Example 2.3, #3(a), #6(b)
- Be able to prove that when a sequence converges, it converges to a unique value (Remark 2.4).
- Know what a *subsequence* is. See #3. Be able to prove Remark 2.6.
- Be able to prove Theorem 2.8.
- Be able to prove facts about convergence, like in #4, 5, 6(a), 8.

• Section 2.2

- Know the Squeeze Theorem (Theorem 2.9). Be able to use it to prove convergence of sequences and to find limits, like in Example 2.10, #1, 2.

- Know and be able to prove Theorem 2.11.
- Know and be able to prove Theorem 2.12. Be able to use Theorem 2.12 to break down complicated looking limits into sums/differences/products/quotients of established limits. See Example 2.13, #2.
- Know the definition of *divergence* to $+\infty$ and $-\infty$ and the corresponding laws governing infinite limits (Theorem 2.15). Try #6.
- Know the Comparison Theorem (Theorem 2.17) and be able to prove it.