

Assignments, April 1

25. (4 pts) Let $f : G \rightarrow H$ be a group homomorphism. Let G' be the group of the fibers of f , where $f^{-1}(h_1) * f^{-1}(h_2) = f^{-1}(h_1 h_2)$. Find a group epimorphism $\pi : G \rightarrow G'$ (describe $\pi(g)$ for $g \in G$ and show that π so defined is an onto group homomorphism).
26. (4 pts) Let $f : G \rightarrow H$ be a group homomorphism. Let G' be the group of the fibers of f , where $f^{-1}(h_1) * f^{-1}(h_2) = f^{-1}(h_1 h_2)$. Find a group monomorphism $\psi : G' \rightarrow H$ (describe $\psi(f^{-1}(h))$ for $f^{-1}(h) \in G'$ and show that ψ so defined is a one-to-one group homomorphism).
27. (4 pts) Let G be a group and let $H \leq G$ and $K \leq G$. Define $HK = \{g \in G \mid g = hk \text{ for some } h \in H \text{ and some } k \in K\}$. Prove that $HK \leq G$ if and only if $HK = KH$.
28. (3 pts) Let G be a finite group of prime order. Prove that G has no non-trivial subgroups.
29. (3 pts) Let G be a group with no non-trivial subgroups. Prove that G is finite, of prime order.
30. (3 pts) Let G be a group and let $H < G$. Let $g \in G$. Prove that $gHg^{-1} = \{ghg^{-1} \mid h \in H\}$ is a subgroup of G and that $|gHg^{-1}| = |H|$.