Assignments, February 6

18. (3 pts) In the definition of a group homomorphism, the sole condition was that $f(g_1g_2) = f(g_1)f(g_2)$. In the definition of a group G acting on a set A, we insisted that $g_1 \cdot (g_2 \cdot x) = g_1g_2 \cdot x$ and that $1_G \cdot x = x$. Why is the seemingly extra condition about the action of 1_G necessary?

19. (3 pts) Let G act on A. We defined the kernel of the group action to be the kernel of the homomorphism $\phi : G \to S_A$ given by $\phi(g) = \sigma_g$ where $\sigma_g(a) = g \cdot a$. The book defined the kernel of the group action to be $\{g \in G | g \cdot a = a \forall a \in A\}$. Show that the two definitions are equivalent.

20. (2 pts) Let G act on itself by conjugation. Show that the kernel of this action is $\{g \in G | gx = xg \forall x \in G\}$. (This is called the center of G and is denoted $Z(G)$).