

Assignments, February 6

Definition 0.1. A homomorphism from a group G to a group H is a function $f : G \rightarrow H$ such that $f(g_1g_2) = f(g_1)f(g_2)$.

13. (3 pts) Prove that if $f : G \rightarrow H$ is a homomorphism then $f(1_G) = 1_H$.
14. (3 pts) Prove that if $f : G \rightarrow H$ is a homomorphism then $f(g^{-1}) = f(g)^{-1}$ for all $g \in G$.
15. (5 pts) If $f : G \rightarrow H$ is a group homomorphism, is $f(G)$ a subgroup of H ? If K is a subgroup of G , is $f(K)$ a subgroup of H ? In the event that you can find a counterexample to either of the above, can you make some extra assumptions to fix things up?
16. (5 pts) If $f : G \rightarrow H$ is a group homomorphism and L is a subgroup of H , is $f^{-1}(L) = \{g \in G \mid f(g) \in L\}$ a subgroup of G ? In the event that you can find a counterexample, can you make some extra assumptions to fix things up?
17. (5 pts) If G is abelian, and $f : G \rightarrow H$ is a group homomorphism, is H abelian? In the event that you can find a counterexample, can you make some extra assumptions to fix things up?