Homework One

Due at the beginning of class, Friday, January 30.

- 1.1.16 Let x be an element of G. Prove that $x^2 = 1$ if and only if $|x|$ is either 1 or 2.
- 1.1.17 Let x be an element of G. Prove that if $|x| = n$ for some positive integer n then $x^{-1} = x^{n-1}$.
- 1.1.31 Prove that any finite group G of even order contains an element of order 2. [Let $t(G)$ be the set $\{g \in G | g \neq g^{-1}\|$. Show that $t(G)$ has an even number of elements and every nonidentity element of $G \setminus t(G)$ has order 2.]
- 1.1.32 If x is an element of finite order n in G, prove that the elements $1, x, x^2, \ldots, x^{n-1}$ are all distinct. Deduce that $|x| \leq |G|$.
- 1.2.2 Use the generators and relations above to show that if x is any element of D_{2n} which is not a power of r, then $rx = xr^{-1}$.
- 1.2.3 Use the generators and relations above to show that every element of D_{2n} which is not a power of r has order 2. Deduce that D_{2n} is generated by the two elements s and sr, both of which have order 2.
- 1.2.7 Show that $< a, b | a^2 = b^2 = (ab)^n = 1 >$ gives a presentation for D_{2n} in terms of the two generators $a = s$ and $b = sr$ of order 2 computed in exercise 3 above. [Show that the relations for r and s follow from the relations for a and b and, conversely, the relations for a and b follow from those for r and s.]