Homework Two

Due at the beginning of class, Monday, February 9.

- (1.3.20) Find a set of generators and relations for S_3.
- (1.5.2) Write out the group tables for S_3, D_8 and Q_8.
- (1.5.3) Find a set of generators and relations for Q_8.
- (1.6.14) Let G and H be groups and let $\varphi : G \to H$ be a homomorphism. Define the *kernel* of φ to be $\{g \in G | \varphi(g) = 1_H\}$ (so the kernel is the set of elements in G which map to the identity of H, i.e. the fiber over the identity of H.) Prove that the kernel of φ is a subgroup of G. Prove that φ is injective (recall this means one-to-one) if and only if the kernel of φ is the identity subgroup of G.
- (1.6.20) Let G be a group and let $Aut(G)$ be the set of all isomorphisms from G onto G. Prove that $Aut(G)$ is a group under function composition (called the *automorphism group* of G and the elements of $Aut(G)$ are called *automorphisms* of G). [You can assume that function composition is associative.]