

Homework Four

Due at the beginning of class, Friday, March 27.

- (2.3.1) Find all subgroups of $\mathbb{Z}/45\mathbb{Z}$, giving a generator for each. Describe the containments between these subgroups.
- (2.3.3) Find all generators for $\mathbb{Z}/48\mathbb{Z}$.
- (2.3.18) Show that if H is any group and h is an element of H with $h^n = 1$, then there is a unique homomorphism from $\mathbb{Z}/n\mathbb{Z}$ to H such that $1 \mapsto h$.
- (2.3.19) Show that if H is any group and h is an element of H , then there is a unique homomorphism from \mathbb{Z} to H such that $1 \mapsto h$.
- (2.3.25) Let G be a cyclic group of order n and let k be an integer relatively prime to n . Prove that the map $x \mapsto x^k$ is surjective. Use Lagrange's Theorem (ex. 19, section 1.7, the fourth problem on homework three) to prove that the same is true for any finite group of order n .