Homework Six

Due at the beginning of class, Monday, April 20.

- (3.1.2) Let $\phi : G \to H$ be a homomorphism of groups with kernel K, and let $a, b \in \phi(G)$. Let $X \in G/K$ be the fiber above a and $Y \in G/K$ be the fiber above b. Fix an element u of X. Prove that if $XY = Z$ in G/K and $w \in Z$ then there is some $v \in Y$ such that $uv = w$.
- (3.2.6) Let $H \leq G$ and let $g \in G$. Prove that if the right coset Hg equals *some* left coset aH of H in G then it equals the left coset gH.
- (3.2.8) Prove that if H and K are finite subgroups of G whose orders are relatively prime then $H \cap K = 1$.
- (3.2.16) Use Lagrange's Theorem in the multiplicative group $(\mathbb{Z}/p\mathbb{Z})^\times$ to prove *Fermat's Little Theorem*: If p is prime then $a^p \cong a(modp)$ for all $a \in \mathbb{Z}$.
- Let $f : G \to H$ be a group epimorphism with kernel K. Prove that there is a one-to-one correspondence between subgroups of H and subgroups of G that contain K such that $K \leq A \leq B$ if and only if $f(A) \leq f(B)$.