

Assignments, March 25

21. (3 pts) Prove that if $G = \langle g \rangle$ is infinite then $G = \langle g^n \rangle$ if and only if $n = 1$ or $n = -1$.
22. (3 pts) Let $G = \langle g \rangle$ be infinite and n, m positive integers with $n \neq m$. Show that $\langle g^n \rangle \neq \langle g^m \rangle$.
23. (3 pts) Let $G = \langle g \rangle$ be infinite and n, m positive integers. State and prove a conjecture regarding under which conditions $\langle g^n \rangle \subseteq \langle g^m \rangle$.
24. (3 pts) Which of these are cyclic (find a generator or prove that G is not cyclic).
 - (a) $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$
 - (b) $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}$
 - (c) $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$