

## Homework Two

1. Verify that  $p \vee (p \wedge q) \Leftrightarrow p$  and  $p \wedge (p \vee q) \Leftrightarrow p$  are tautologies.
2. For integers  $x$  and  $y$ , find the inverse, the converse, the contrapositive, and the negation of each of the following statements.
  - (a) If  $x = 3$ , then  $x^4 = 81$ .
  - (b) If  $x > 0$ , then  $x \neq -4$ .
  - (c) If  $x$  is odd and  $y$  is even, then  $xy$  is even.
  - (d) If  $x^2 = x$ , then either  $x = 0$  or  $x = 1$ .
  - (e) If  $xy \neq 0$ , then  $x \neq 0$  and  $y \neq 0$ .
  - (f) If  $x - 3 = 0$  or  $x - 2 = 0$  then  $x^2 - 5x + 6 = 0$ .
3. Negate the following statements:
  - (a) There exists a real number  $x$  such that  $x^2 = 2$ .
  - (b) No rational number  $x$  is such that  $x^2 = 2$ .
  - (c) For all real numbers  $x$  either  $|x| = x$  or  $|x| = -x$ .