

Homework Two

1. Prove that the sum of any two consecutive natural numbers, plus the larger of the two numbers, results in a perfect square.

Definition 1. A natural number n is even if $2|n$. If n is not even we say that n is odd.

2. Prove that a natural number n is even if and only if $n + 1$ is odd.
3. Prove directly that if m and n are odd then mn is odd.
4. Prove by contradiction that if m and n are odd then mn is odd.
5. Prove that if every even natural number greater than 2 is the sum of two primes then every odd natural number greater than 5 is the sum of three primes.

Your illustrious professor has decided to undertake a serious scientific study of “free time” for the remainder of the semester. In order to free up some time to do this, he has left the chore of grading (certainly one of a math professor’s most thankless undertakings) to you. You are to use the following scale:

- Assign a grade of R (= right) if the claim and proof are correct (even if the proof is not the simplest or most elegant or shortest or the one you would have given).
- Assign a grade of W (= wrong) if the claim is incorrect, or if the main idea in the alleged proof given is incorrect, or if most of the statements contained within the alleged proof are incorrect.
- Assign a grade of B (= in between) for a proof that is largely correct but contains one or two incorrect statements or justifications.

Students, of course, like to have feedback on their work, so whenever you assign a grade of W or B , you need to explain your grade. Tell what is incorrect and why.

6. *Claim* If m^2 is odd then m is odd.

Proof Assume m is odd. Then $m = 2k + 1$ for some integer k . From this we have that $m^2 = (2k + 1)^2 = 4k^2 + 4k + 1$. Factoring a 4 out of the first two terms we have $m^2 = 4(k^2 + k) + 1 = 2(2(k^2 + k)) + 1$, which is odd. Thus if m^2 is odd then m is odd. \square

7. *Claim* If $a|b$ and $a|c$ and $b > c$ then $a|(b - c)$.

Proof Assume that $a|b$ and $a|c$ but $a \nmid (b - c)$. Then there is no natural number q such that $aq = (b - c)$. Now since $a|b$ there is an m such that $am = b$ and since $a|c$ there is an n such that $an = c$. Thus $b - c = am - an = a(m - n)$. Thus we have that $a|(b - c)$. But we assumed that $a \nmid (b - c)$. Here we have a contradiction, and we are forced to conclude that if $a|b$ and $a|c$ and $b > c$ then $a|(b - c)$. \square