Homework 6

Due at the beginning of class on Monday, March 22.

Definition 1. *Given sets A and B, and a relation $R \subseteq A \times B$, define the relation $R^{-1} \subseteq B \times A$ by $(b, a) \in R^{-1}$ if $(a, b) \in R$.*

Definition 2. *Given sets A and B, and a relation $R \subseteq A \times B$, we say that the relation R is* onto *if for every $b \in B$ there is some $a \in A$ with $(a, b) \in R$.*

1. Let $R \subseteq A \times A$ be a partial order relation (i.e. R is reflexive, antisymmetric, and transitive) on a set A. Prove that R^{-1} is also a partial order relation on A.

2. Prove or give a counterexample to each of the following:

   (a) If $R \subseteq A \times A$ and $S \subseteq A \times A$ are transitive relations then $R \cup S$ is a transitive relation.

   (b) If $R \subseteq A \times A$ and $S \subseteq A \times A$ are transitive relations then $R \cap S$ is a transitive relation.

3. If $R \subseteq A \times B$ and $S \subseteq B \times C$ are relations, define the relation $S \circ R \subseteq A \times C$ by $(a, c) \in S \circ R$ if there is $b \in B$ with $(a, b) \in R$ and $(b, c) \in S$.

   (a) Prove that if R and S are both onto then $S \circ R$ is onto.

   (b) Prove that if $S \circ R$ is onto then S is onto.

   (c) Give an example to show that if $S \circ R$ being onto need not imply that R is onto.