Definition 1. *Let $f : A \to B$ be a function and let $T \subseteq B$. Define $f^{\hookleftarrow}(T) = \{a \in A | f(a) \in f(T)\}$. Note that sometimes $f^{-1}(T)$ is used instead to denote this, but can be confusing as we are not referring to an inverse function.*

1. Let $f : A \to B$ be a function and let $S \subseteq A$. Prove that $S \subseteq f^{\hookleftarrow}(f(S))$ and give an example showing that this containment can be proper (i.e. $S \subset f^{\hookleftarrow}(f(S))$).

2. What condition(s) are needed on f to ensure that $S = f^{\hookleftarrow}(f(S))$? State and prove your result.

3. Let f be a function from a set A to a set B, and let S and T be subsets of A.

   (a) If f is onto is it the case that $f(S \cap T) = f(S) \cap f(T)$? Prove this or give a counterexample.

   (b) If f is one-to-one is it the case that $f(S \cap T) = f(S) \cap f(T)$? Prove this or give a counterexample.

4. For each element of S_5, draw a function diagram indicating where each element (1-5) goes:

   (a) $(132)(42)(541)(23)$

   (b) $(23)(34)(45)$

   (c) $(14)(13)(12)$

5. Consider the function f from the open interval $(0, 1)$ to the set of real numbers $\mathbb{R}$ defined by the following formula:

$$f(x) = \begin{cases} \frac{x-1/2}{x} & \text{for } x \leq 1/2 \\ \frac{x-1/2}{1-x} & \text{for } x > 1/2. \end{cases}.$$

   Prove that f is one-to-one and onto.