Homework Two

1. Verify that $p \vee (p \wedge q) \Leftrightarrow p$ and $p \wedge (p \vee q) \Leftrightarrow p$ are tautologies.

p	q	r	$(p \wedge q)$	$p \vee (p \wedge q)$	$(p \vee q)$	$p \wedge (p \vee q)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	F	T	F
F	T	F	F	F	T	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Notice that the columns under $p \vee (p \wedge q)$ and $p \wedge (p \vee q)$ are identical to the column under p.

2. For integers x and y, find the inverse, the converse, the contrapositive, and the negation of each of the following statements.

(a) If $x = 3$, then $x^4 = 81$.

inverse:	If $x \neq 3$ then $x^4 \neq 81$
converse:	If $x^4 = 81$ then $x = 3$
contrapositive:	If $x^4 \neq 81$ then $x \neq 3$.
negation:	$x = 3$ and $x^4 \neq 81$.

(b) If $x > 0$, then $x \neq -4$.

inverse:	If $x \leq 0$ then $x = -4$.
converse:	If $x \neq -4$ then $x > 0$.
contrapositive:	If $x = -4$ then $x \leq 0$.
negation:	$x > 0$ and $x = -4$.

(c) If x is odd and y is even, then xy is even.

inverse:	If x is even or y is odd then xy is odd.
converse:	If xy is even then x is odd and y is even.
contrapositive:	If $x = -4$ then $x \leq 0$.
negation:	$x > 0$ and $x = -4$.

(d) If $x^2 = x$, then either $x = 0$ or $x = 1$.

inverse:	If $x^2 \neq x$ then $x \neq 0$ and $x \neq 1$.
converse:	If $x = 0$ or $x = 1$ then $x^2 = x$.
contrapositive:	If $x \neq 0$ and $x \neq 1$ then $x^2 \neq x$.
negation:	$x^2 = x$ and $x \neq 0$ and $x \neq 1$.

(e) If $xy \neq 0$, then $x \neq 0$ and $y \neq 0$.

inverse:	If $xy = 0$ then $x = 0$ or $y = 0$
converse:	If $x \neq 0$ and $y \neq 0$ then $xy = 0$.
contrapositive:	If $x = 0$ or $y = 0$ then $xy = 0$.
negation:	$xy \neq 0$ but $x = 0$ or $y = 0$.

(f) If $x - 3 = 0$ or $x - 2 = 0$ then $x^2 - 5x + 6 = 0$.

inverse:	If $x - 3 \neq 0$ and $x - 2 \neq 0$ then $x^2 - 5x + 6 \neq 0$.
converse:	If $x^2 - 5x + 6 = 0$ then $x - 3 = 0$ or $x - 2 = 0$.
contrapositive:	If $x^2 - 5x + 6 \neq 0$ then $x - 3 \neq 0$ and $x - 2 \neq 0$.
negation:	$x - 3 = 0$ or $x - 2 = 0$ and $x^2 - 5x + 6 \neq 0$.

3. Negate the following statements:

   (a) There exists a real number x such that $x^2 = 2$.
   For all real numbers x, $x^2 \neq 2$.

   (b) No rational number x is such that $x^2 = 2$.
   There is a rational number x such that $x^2 = 2$.

   (c) For all real numbers x either $|x| = x$ or $|x| = -x$.
   There is a real number x such that $|x| \neq x$ and $|x| \neq -x$.