① Let $\sim \subseteq Z \times Z$ be the relation define by $x \sim y$ if $5 \mid (y-x)$

So, $R \subseteq \{(x,y) \in Z \times Z \mid 5 \mid (y-x)$ or

$R \subseteq \{(x,y) \in Z \times Z \mid 5$ divides $(y-x)\}$

thus,

$$Z \times Z = \{(-5,5), (0,5), (6,6), (5,-5), \ldots, (2,12), (15,5), (7,2), \ldots, (2,7), (-5,10), (15,5), \ldots, (5,15), (12,2), (20,-20), (-20,20), \ldots, (5,0), (8,8), (10,-10), (-10,10), \ldots,\}$$

then,

$$R = \{(-5,5), (5,-5), (0,5), (2,12), (12,2), (15,5), (5,15), (7,2), (2,7), (-5,10), (10,-5), \ldots,\}$$

(a) Prove that $\sim$ is reflexive and symmetric.

Proof: Let be a relation on a set $Z$, then $R$ is said to be,

Reflexive if $aRa\ \forall\ a \in Z$ i.e. if $(a,a) \in R\ \forall\ a \in Z$

So the $\sim \subseteq Z \times Z$ is not Reflexive.

and

Symmetric, if $aRb \Rightarrow bRa$ i.e., if $(a,b) \in R \Rightarrow (b,a) \in R$

~~Da~~, the $\sim \subseteq Z \times Z$ is symmetric

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