Eric Negem

(3) Prove that for any a and b in A either $[a]=[b]$ or $[a]\cap[b]=\varnothing$.

Let $R \subseteq A \times A$ be an equivalence relation and define $[a]$ to be $\{b \in A \mid a \sim b\}$ and $[b]$ to be $\{a \in A \mid b \sim a\}$. If $b \in [a]$ we know that $a \sim b$ and since R is symmetric we also know that $b \sim a$, so $a \in [b]$. So $[a] = \{b_1, b_2, b_3 \dots b_n\}$ and $[b] = \{a_1, a_2, a_3 \dots a_n\}$. Thus if $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3 \dots a_n = b_n$ then $[a]=[b]$ and if $a_1 \neq b_1$, $a_2 \neq b_2$, $a_3 \neq b_3 \dots a_n \neq b_n$ then $[a]\cap[b] = \varnothing$. Therefore for any a and b in A either $[a]=[b]$ or $[a]\cap[b]=\varnothing$.