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Foundations Math 3425-001  
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## Participation 4a

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Think about the equivalence relation  $\sim$  on  $\mathbb{Z}$  given by  $x \sim y$  if  $5|(y-x)$ . Prove that if  $x \sim y$  and  $a \sim b$  then  $x+a \sim y+b$ .

**Proof:** Suppose  $x \sim y$  and  $a \sim b$ . We will prove that  $x+a \sim y+b$  when  $x \sim y$  is  $5|(y-x)$ . Let  $a, b, q, r, x, y$  be  $\mathbb{Z}$ . There exists a  $q$ , such that  $5q = y - x$ . Then  $5q + x = y$ .

There exists a  $r$  such that  $5r = b - a$ . Then  $5r + a = b$ .

Thus  $y + b = 5q + x + 5r + a$

or  $y + b = 5q + 5r + x + a$

or  $(y + b) = (5q + 5r) + (x + a)$

rearranging  $(y + b) - (x + a) = 5(q + r)$

Therefore  $x+a \sim y+b$ .