

## Foundations Presentation

$$A \quad B \\ \{1, 2, 3\} \quad \{4, 5, 6\}$$

$$f: A \rightarrow B = \{(1, 4), (2, 5), (3, 6)\}$$

$$g: A \rightarrow B = \{(1, 5), (2, 6), (3, 4)\}$$

$$h: A \rightarrow B = \{(1, 6), (2, 4), (3, 5)\}$$

$$i: A \rightarrow B = \{(1, 4), (2, 6), (3, 5)\}$$

$$j: A \rightarrow B = \{(1, 5), (2, 4), (3, 6)\}$$

$$k: A \rightarrow B = \{(1, 6), (2, 5), (3, 4)\}$$

$$S = \{f, g, h, i, j, k, l\}$$

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Proof: First, we need  $\sim$  to be reflexive, in other words, we need  $f \sim f$  for any  $f \in S$ .

For  $f \sim f$ ,  $f$  would have to reverse the same number of orientations as  $f$ , which it does

so  $f \sim f$  and  $\sim$  is reflexive. Next, let

$f \sim g$  for any  $f, g \in S$ . Then  $f$  reverses the same number of orientations as  $g$ , so clearly

$g \sim f$  and  $\sim$  is symmetric. Finally, let  $f \sim g$

and  $g \sim h$  with  $f, g, h \in S$ . Since  $f \sim g$ ,  $f$  and

$g$  reverse the same number of orientations. Also,

$g$  and  $h$  reverse the same number of orientations.

Since  $f$  and  $g$  reverse the same number of

orientations and  $g$  and  $h$  reverse the same

number of orientations,  $f$  and  $h$  reverse the

same number of orientations, thus  
 $f \sim h$  and  $\sim$  is transitive. Now, we have  
 $\sim$  reflexive, symmetric, and transitive, so  
 $\sim$  is an equivalence relation.  $\square$

### Equivalence Classes $\circ$

0 reversals  $\rightarrow \{(1, 4), (2, 5), (3, 6)\}$

1 reversal  $\rightarrow \{(1, 4), (2, 6), (3, 5)\}$   
 $\{(1, 5), (2, 4), (3, 6)\}$

2 reversals  $\rightarrow \{(1, 5), (2, 6), (3, 4)\}$   
 $\{(1, 6), (2, 4), (3, 5)\}$

3 reversals  $\rightarrow \{(1, 6), (2, 5), (3, 4)\}$