1. Let R is a relation on a set A such that for all $a \in A$, aRx for some $x \in A$ (i.e. everybody is related to somebody). Prove that if R is symmetric and transitive, then R is reflexive.

2. Consider the relation R on the real numbers $\mathbb{R}$ given by aRb when $|a-b| < \epsilon$ for some fixed positive number ϵ. Is R reflexive? Is R symmetric? Is R transitive?

3. Consider the set S of all one-to-one and onto functions from the set $\{1, 2, 3\}$ to the set $\{4, 5, 6\}$. For example f $(f(1) = 6, f(2) = 5, f(3) = 4)$ and g $(g(1) = 5, g(2) = 4, g(3) = 6)$. Write down all such functions and prove that your list is exhausive (i.e. that you have indeed written down all of them).

4. Consider the set S above. Note that for any $f \in S$, for $i < j$ either $f(i) < f(j)$ or $f(i) > f(j)$ since $f(i) = f(j)$ is impossible due to the one-to-one property. If $f(i) < f(j)$ we say that f preserves the orientation of i and j. If $f(i) > f(j)$ we say f reverses the orientation of i and j. Define $\sim$ on S as follows: $f \sim g$ if f reverses the same number of orientations (out of all possible pairs i, j as g). Prove that f is indeed an equivalence relation and write down the equivalence classes.