

S is the set of all 1-1 and onto functions from $\{1, 2, 3\}$ to $\{4, 5, 6\}$

List all possible functions

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 4 & 6 & 5 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 5 & 4 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 5 & 6 & 4 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 6 & 4 & 5 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \end{pmatrix}$$

Prove that they are exhaustive

Since the inputs have 3 elements and all the functions are one-to-one and onto, then all out-puts must have 3 elements

If we start with $S(1)$ then $S(1)$ can be mapped one of 3 elements. If we choose $S(2)$ next it can only be mapped to 2 elements, ect.

⑥ Multiplying the # of choices yields $3 \cdot 2 \cdot 1 = 6$ which is the # of functions from $\{1, 2, 3\}$ to $\{4, 5, 6\}$ that are both one-to-one and onto.