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#2 Consider the relation R on the real numbers $\mathbb{R}$ given by aRb when $|a-b| < \epsilon$ for some fixed positive # ϵ. Is R reflexive? Is R symmetric? Is R transitive?

Reflexive:

There exists an $|a-a| < \epsilon$ where $0 < \epsilon$ and zero is a small positive number. Thus it is reflexive. $\square$

Symmetric:

We know that aRb when $|a-b| < \epsilon$. By definition of absolute value we know $|a-b|$ will always be positive numbers. If we change the order $|-b+a| < \epsilon$ we still get a positive number. Thus $|a-b| < \epsilon$ is symmetric and aRb, bRa. $\square$

Transitive:

Counter Example:

$a = 0 \quad c = {}^-2/3$

$b = 1/3 \quad \epsilon = 1/2$

$$|0 - 1/3| < 1/2$$

$$\left|\frac{1}{3} + \frac{2}{3}\right| < \frac{1}{2}$$

This shows that $|a-b| < \epsilon$ is not transitive.