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3.) CONSIDER $f: A \rightarrow B$ AND $g: B \rightarrow A$. PROVE OR GIVE A COUNTEREXAMPLE.
IF $g(f(a)) = a$ FOR ALL $a \in A$, THEN $f(g(b)) = b$ FOR ALL $b \in B$.

PROOF:

LET $g(f(a)) = a$ FOR ALL $a \in A$. SINCE $f: A \rightarrow B$, WE KNOW THAT $(a, b) \in f$ FOR ONLY ONE $b \in B$. SIMILARLY, SINCE $g: B \rightarrow A$ WE KNOW THAT $(b, a) \in g$ FOR ONLY ONE $a \in A$. WITHOUT LOSS OF GENERALITY, $f(a) = b$, AND $g(b) = a$, AND $g(f(a)) = g(b) = a$. THEN $f(g(b)) = f(a) = b$. THUS, $f(g(b)) = b$ FOR ALL $b \in B$. \square