1. Let $f : A \to A$ be a function. Prove that if $f \circ f$ is injective then f is injective.

2. Consider $f : A \to B$ and $g : B \to A$. Prove or give a counterexample: If $f(g(b)) = b$ for all $b \in B$ then f is one-to-one and onto.

3. Consider $f : A \to B$ and $g : B \to A$. Prove or give a counterexample: If $g(f(a)) = a$ for all $a \in A$, then $f(g(b)) = b$ for all $b \in B$.

4. Let $\mathbb{N}^+ = \{1, 2, 3, 4, \ldots\}$ (some authors think of this as $\mathbb{N}$, while others think of this as $\mathbb{N} \setminus \{0\}$). Which of the following define onto functions from $\mathbb{N}^+ \times \mathbb{N}^+$ to $\mathbb{N}^+$?

   (a) $f((a, b)) = a + b$

   (b) $g((a, b)) = ab$

   (c) $h((a, b)) = ab(b + 1)/2$

5. Let $f : \mathbb{R} \to \mathbb{R}$. Suppose that there is a positive real numbers c and n such that $|f(x) - f(y)| \geq c|x - y|^n$ for all $x, y \in \mathbb{R}$. PRove that f is one to one.