

④ Consider the set $\{0, 1, 2, 3, 4\}$ and the operation

$$x * y = 2y - x \pmod{5}$$

Write a Cayley table for this operation.

Is it commutative?

Is it associative?

Cayley Table

$$x * y = 2y - x$$

$$-1 \pmod{5} = -1$$

$2y - x \pmod{5}$	0	1	2	3	4
0	0	2	4	1	3
1	-1	1	3	0	2
2	-2	0	2	4	1
3	-3	-1	1	3	0
4	-4	-2	0	2	4

Commutativity

* is not commutative
for a) it is not reflexive across the diagonal.

$$b. x * y \neq y * x \quad \forall x, y \in \{0, 1, 2, 3, 4\}$$

$$\text{with } x=1, y=2 \Rightarrow x * y = 3$$

$$y * x = 0$$

$$0 \neq 3 \text{ Hence}$$

Not commutative

Associativity $\Rightarrow \forall r, s, t$ we have
 $(r * s) * t = r * (s * t)$

$$\text{Let } x=1, y=2$$

$$(2y - x) \pmod{5} = 2y - (x \pmod{5})$$

Since $x \in \{0, 1, 2, 3, 4\}$ $x \pmod{5} = x$ since $x < 5$

$$\text{Let } y=4, x=0$$

$$(8 - 0) \pmod{5} = 8 \pmod{5} = 3$$

$$8 - (0 \pmod{5}) = 8 - 0 = 8 \neq 3$$

Hence not associative.