1. Let $S = \{0123456\}$, and consider the operation $\cdot$ on S given by $x \cdot y = xy \mod 7$. Write a Cayley table for this operation. Is there an identity element? If so, does every element have an inverse?

2. Let A be a set and consider the operation $\heartsuit$ on $\mathcal{P}(A)$ given by $S \heartsuit T = (S \setminus T) \cup (T \setminus S)$.

   (a) Prove that $\heartsuit$ is a binary operation on $\mathcal{P}(A)$.

   (b) Is $\heartsuit$ associative? (prove or give a counterexample)

   (c) Is $\heartsuit$ commutative? (prove or give a counterexample)

   (d) Is there an identity element for $\heartsuit$ (if so – prove that there is, if not give a counterexample of a set A for which $\heartsuit$ has no identity element)? If there is always an identity element for $\heartsuit$, for any $B \subseteq A$ is there an inverse B^{-1} such that $B \heartsuit B^{-1}$ equals this identity element? (prove or give a counterexample)

3. Prove that there are no onto functions from $[4]$ to $[5]$. (Recall that $[n] = \{1, 2, 3, \ldots, n\}$.)

4. Consider the set $\{0, 1, 2, 3, 4\}$ and the operation $*$ on it given by $x * y = 2y - x \mod 5$. Write a Cayley table for this operation. Is it commutative? Is is associative?