Homework One

1. In class we calculated the moment generating function for $Y \sim N(0, \sigma^2)$ to be $M_Y(t) = e^{\frac{\sigma^2 t^2}{2}}$. Use this (or do it the hard way if you like) to compute $M_X(t)$ for $X \sim N(\mu, \sigma^2)$. [HINT: What is the relationship between X and Y?]

2. Consider these facts from class:

   - $P(-1.96 < Z < 1.96) = \int_{-1.96}^{1.96} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz = .95$ for $Z \sim N(0, 1)$.
   - If $X_1, X_2, \ldots, X_n$ are independent normal $N(\mu, \sigma^2)$ random variables then $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$.
   - If $Y \sim N(\mu, \sigma^2)$ then $\frac{Y-\mu}{\sigma} \sim N(0, 1)$.

   Fill in ? and ?? in the following expression:

   $$P(? < \bar{x} - \mu < ??) = .95$$

   (of course derive your solution). Use your answer to fill in ? and ?? in the following:

   $$P(? < \mu < ??) = .95$$

   (i.e. solve for μ).